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OPTIMAL BARGAINING AND MORAL HAZARD

Abstract. In this paper we attempt to find the answer to the question of how the professional boxers behave *ex post* to additional money share negotiated *ex ante*. We find that for each fighter under a certain stake threshold there is an incentive to put in huge efforts for the forthcoming fight and that above that threshold, the opposite is true leading to a phenomenon of moral hazard. We also find that bargaining is successful in the absence of moral hazard. Finally, the optimal effort of the fighter will increase (decrease) for his (his opponents') additional outside option.

Keywords: bargaining, boxing, efforts, incentives, moral hazard, threshold share

1. Introduction

Despite the important position of professional boxing analysis in the area of sports economics, the number of articles using the tools of economic theory explaining the incentives and corresponding behavior of the professional boxers is still limited. A number of interesting questions has been raised and answered by various microeconomic tools such as contract (Tenorio, 2000), contest (Amegashie and Kutsoati, 2005; Akin et al., 2022[1]), bargaining (Issabayev and Oskenbayev, 2019; Akin et al., 2022), and several extensions for future works have been suggested. Our paper is a contribution to one of the extensions in the literature on the link between money share from negotiation and efforts of the fighters afterwards.

So, one of the research questions worth considering is what will happen to the efforts of the pro boxers preparing for the forthcoming fight once their money purse is guaranteed *ex ante*. Put it differently, *will the guaranteed money amount create incentives for the fighters to train hard to win the fight?* In fact, this is a very important question. At first, one may say that financial incentives certainly inspire fighters to work harder for, at least, to make a name for the case of a challenger or to retain his popularity for the case of a champion. This is mostly true in the eyes of naïve fans though the efforts of the fighters are not observable. However, it is obvious that fighters receive their purse after the bout as signed in the contract negotiated regardless of the outcome. In other words, fighters are paid for risking their health (life) to fight in the ring for 12 rounds with a maximum duration of one hour rather than for the amount of effort put during the training, which could last from 3 to 6 months. An interesting feature is that the fans usually do not value true efforts of the fighters. Hence, it shouldn't be surprising that the fighters, by hiding their actions, could be putting less effort in training upon learning their guaranteed purse (Akin et al., 2022; Tenorio, 2000; Amegashie and Kutsoati, 2005). This idea has prompted our paper to analyze how the behavior of pro boxers changes for an additional money stake.

Unlike a three-stage model in Akin et al (2022) where the challengers are chosen endogenously, here we rather propose a two-stage model assuming that the challenger is already chosen exogenously provided the different issues of interest in this paper. In the first stage, the two Boxers negotiate on how to share money revenue collected according to Nash bargaining framework like in Issabayev and Oskenbayev (2019). In the second stage, contingent on the money share from the first stage, the fighters maximize their payoffs by choosing their optimal efforts during the

[1] Please see Akin et al. (2022) build a three-stage model, where in the first stage the chooser selects one of the challengers, then in the second stage they negotiate on money purse sharing, and finally, in the third stage each fighter chooses their optimal efforts. But, they didn't manage to build a link between the last two stages that the current paper has tried to do so.

preparation for the upcoming fight. Thus, the efforts of the fighters should virtually depend on the amount of money share they are to receive as negotiated rather than the vice versa.

We now offer a brief summary of our main results. When forward looking fighters choose their efforts in the second stage, we show that the optimal efforts of the fighters are a function of the endogenous money share negotiated and the exogenous share of money revenue collected from the respective fans of the fighters[2]. Consequently, we show in Propositions 1 and 2 that the reactions of fighters in response to additional money share are mixed. We find that financial rewards for each fighter create both incentives and disincentives depending on certain stake thresholds. The latter (disincentives) refers to a moral hazard phenomenon. Combining the outcomes of Proposition 1 and 2, we obtained three different scenarios for the ranges of money share among the fighters and the incentives and disincentives of the fighters under each case. This let us develop Proposition 3 demonstrating that the bargaining equilibrium on money share is reached in the absence of moral hazard by each fighter. We also find that with the absence of moral hazard by each fighter, the increase in the shares of the money revenue contributed by the fans of champs motivates (demotivates) the champ (challenger) to put more efforts in preparation for the upcoming fight. Finally, we find that the outside options of the fighters have an indirect impact on the fighters' efforts. In the absence of moral hazard, an additional outside option of the fighter (opponent), which is assumed to be part of his bargaining power a la Issabayev and Oskenbayev (2019), has positive (negative) impact on his money share, which in turn increases (decreases) his optimal efforts.

The rest of the paper is organized as follows. Section 2 describes a two-stage model along with its outcomes. Section 3 concludes the paper.

2. The Model

Once again our model consists of two stages. In stage 1, the two fighters negotiate on money sharing according Nash bargaining framework like in Issabayev and Oskenbayev (2019), so that $\boldsymbol{\beta}$ is the fraction of money revenue, $\boldsymbol{M}$, to the current Champion (1) and the rest $\mathbf{1}-\boldsymbol{\beta}$ belongs to the Challenger (2). The utility of the payoff from bargaining, $\boldsymbol{U_i(\beta)}$ for $\boldsymbol{i=1,2,}$ for each fighter is linear. Then,

$$\max_{\beta\epsilon(\beta_L,\beta_H)} [\boldsymbol{\beta M - d_1}][(\mathbf{1}-\boldsymbol{\beta})\boldsymbol{M - d_2}] \qquad [1]$$

Note for the bargaining to hold each fighter's purse must be greater than his outside option ($\boldsymbol{d_1 > 0}$ and $\boldsymbol{d_2 > 0}$). That is, the incentive compatible conditions are $\boldsymbol{\beta M - d_1 > 0}$ and $(\mathbf{1}-\boldsymbol{\beta})\boldsymbol{M - d_2 > 0}$. Otherwise the fighters would walk away. This, in turn, leads to $\boldsymbol{M - d_1 - d_2 > 0}$ so that the total revenue from the upcoming fight is predicted to be larger than the outside options of both fighters. For each fighter to give up his outside options, the following minimum shares must be guaranteed, which are $\boldsymbol{\beta_L > \frac{d_1}{M}}$ to the champ and $\mathbf{1}-\boldsymbol{\beta_H > \frac{d_2}{M}}$ to the challenger, respectively, with $\mathbf{0} < \boldsymbol{\beta_L < \beta_H} < \mathbf{1}$[3]. This implies that the champion wouldn't agree to fight if he were to receive less than $\boldsymbol{\beta_L}$. On the other hand, the champ cannot bargain for more than $\boldsymbol{\beta_H}$ in which case the challenger would walk away. Thus, the optimal share for the champ is predicted to be within the range of $(\boldsymbol{\beta_L, \beta_H})$.

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[2] It is important to note that the money revenue collected from the fans and other sources are not distributed just among the boxers themselves. The other beneficiaries of the organized fight are their respective promoters, managers, coaches, doctors etc. (Issabayev and Oskenbayev, 2019). To get insight of the research question in the current paper, we restrict the beneficiaries of the money revenue to two fighters only.

[3] The subscripts "$\boldsymbol{L}$" and "$\boldsymbol{H}$" stand for *low* and *high* levels, respectively.

We specify the total money revenue[4] for the fight in professional boxing to be $\boldsymbol{M} = (\boldsymbol{e_1 V_1})^{\boldsymbol{\alpha}}(\boldsymbol{e_2 V_2})^{\boldsymbol{1-\alpha}}$. Here the money revenue is jointly determined by the value of titles (or belt) for each player ($\boldsymbol{V_1} > \boldsymbol{0}$ and $\boldsymbol{V_2} > 0$)[5], and their respective associated current efforts ($\boldsymbol{e_1} > 0$ and $\boldsymbol{e_2} > \boldsymbol{0}$). While the belts can serve as *proxies for fighters' popularity,* their current efforts seem to demonstrate a visibly promising "*spectacular*" fight by more dominant boxers (Butler et al, 2020; Chaplin et al, 2017) in the eyes of "naïve" fans. However, on the other hand, the very best boxers rarely fight each other (Tenorio, 2006). In short, we argue that the popularities and the current efforts of the fighters are indivisible in generating money revenue. Put it differently, there is an interaction between the fighters' current efforts and titles in fundraising. Besides, Tenorio (2000) remarked that the fighter's purse is linked to his past performance rather than his contemporaneous one. We concur with him since none of the fighters can become famous or popular overnight. In support of it, the values in money function, $\boldsymbol{V_1}$ and $\boldsymbol{V_2}$, can also be interpreted as the marketability of the fighters (Chaplin, 2012) accumulated from their past performance (efforts). However, the current efforts cannot be completely independent of past performance. Hence, we argue that the current efforts fighters put during the training, watched by fans both online and offline, should also contribute to the money revenue by boosting the promising spectacular outcome[6]. The parameters $\boldsymbol{0 < \alpha < 1}$ and $\boldsymbol{1 - \alpha}$ are the shares of revenue contributed by the respective fans of each fighter.

In stage 2, the fighters choose their optimal efforts to maximize their payoff. The standard cost function of efforts for each boxer is given by $\boldsymbol{C_i(e_i)} = \frac{e_i^2}{2}$ for $\boldsymbol{i = 1, 2,}$ which is subtracted from their respective shared revenue from bargaining. For simplicity, we let the money revenue be $\boldsymbol{M(e_1, e_2) = V e_1^{\alpha} e_2^{1-\alpha}}$ where $\boldsymbol{V = V_1^{\alpha} V_2^{1-\alpha} = 1}$ without loss of generality. Then each player maximizes his payoff as:

$$\max_{e_1} \boldsymbol{\beta e_1^{\alpha} e_2^{1-\alpha}} - \frac{e_1^2}{2} \qquad [2]$$

$$\max_{e_2} (\boldsymbol{1-\beta}) \boldsymbol{e_1^{\alpha} e_2^{1-\alpha}} - \frac{e_2^2}{2} \qquad [3]$$

The first order condition (FOC) in stage 2 will yield us:

$$\boldsymbol{\alpha\beta} \left(\frac{\boldsymbol{e_2}}{\boldsymbol{e_1}}\right)^{\boldsymbol{1-\alpha}} = \boldsymbol{e_1} \qquad [\mathbf{4}]$$

$$(\boldsymbol{1-\alpha})(\boldsymbol{1-\beta}) \left(\frac{\boldsymbol{e_1}}{\boldsymbol{e_2}}\right)^{\boldsymbol{\alpha}} = \boldsymbol{e_2} \qquad [\mathbf{5}]$$

[4] It is important to notice that the money revenue in fact comes from multiple sources: sponsors, PPV sales, popularity of the fighters and so on. As Tenorio (2000) noted that the market value of the fight is largely determined by fighters' reputations. We treat the popularity, which is built over time with a couple of good performances, to be synonymous with the already built reputation like in Akin et al. (2022).

[5] Issabayev and Oskenbayev (2019) specified as $\boldsymbol{V_1 > V_2 > 0}$ due to reputation effect as the fighters value the championship belts differently. However, here the challenger may also be a belt-holder but with a lower division, say IBF, whereas the absolute champion is a high category belt-holder such as WBO or WBA.

[6] Efforts most of the time are unobservable for fans. But to collect more revenue from fans, the promoters of the fighters do their best to organize spectacular fights. That is, they are showing the videos of their training online or offline in the public arena like Madison Square in New York City. Therefore, we assume the amount of money revenue is dependent on fighters' current efforts as well.

The LHS (RHS) terms in [4] and [5] are marginal revenues (costs) of efforts of the fighters. Solving them simultaneously we receive:

$$\boldsymbol{e_1^*(\alpha,\beta) = [\alpha\beta]^{\frac{1+\alpha}{2}}[(1-\alpha)(1-\beta)]^{\frac{1-\alpha}{2}}} \qquad [6]$$

$$\boldsymbol{e_2^*(\alpha,\beta) = [\alpha\beta]^{\frac{\alpha}{2}}[(1-\alpha)(1-\beta)]^{1-\frac{\alpha}{2}}} \qquad [7]$$

The optimal effort of each player is a function of their respective money shares as negotiated in the first stage, and the fractions of money revenue collected from their fans. In simple words, the fighter's decision on how much effort to put in during the training depends on the number of fans willing to watch the fight and the negotiated share. Pushing back to the early question of what will happen to the fighters' efforts in response to additional money share, we obtain the following comparative statics:

$$\boldsymbol{\frac{\partial e_1^*}{\partial \beta} = \frac{e_1^*}{2}\left[\frac{(1+\alpha-2\beta)}{\beta(1-\beta)}\right]\begin{cases} > 0 \text{ if } \beta < \bar{\beta} \\ < 0 \text{ if } \beta > \bar{\beta} \end{cases}} \qquad [8]$$

The $\boldsymbol{\bar{\beta} = \frac{1+\alpha}{2}}$ is the threshold level of money stake at which the current champion is indifferent whether to put in huge or less effort. Similarly,

$$\boldsymbol{\frac{\partial e_2^*}{\partial \beta} = \frac{e_2^*}{2}\left[\frac{(\alpha-2\beta)}{\beta(1-\beta)}\right]\begin{cases} > 0 \text{ if } \beta < \underline{\beta} \\ < 0 \text{ if } \beta > \underline{\beta} \end{cases}} \qquad [9]$$

The $\boldsymbol{\underline{\beta} = \frac{\alpha}{2}}$ is the threshold share that the challenger is willing to offer to the champ at which the challenger is indifferent whether to put in huge or less effort.

Proposition 1: *If the share of the revenue from bargaining in the first stage for the current champ is lower (higher) than the threshold level that he wishes to receive, then the champ is willing to put more (less) efforts to win the fight for additional stakes.*

The proposition1 implies that there is a range of money shares that creates an incentive for the current champ to win the forthcoming fight and retain his title that should have an influence on his future money revenues[7]. On the other hand, once the current champ is already guaranteed to get a larger share from bargaining than he is willing to receive, then it changes his behavior (effort) in the opposite way for additional stakes from the money revenue. Consequently, the probability of a loss from a forthcoming fight is unavoidable. This is the issue of moral hazard that can serve as a testimony to fight when for example the former champion James "Buster" Douglas met Evander Holyfield in early 1990s (Tenorio, 2000) or the most recent case of Saul "Canelo" Alvarez bout against Dmitri Bivol in May 2022. Since both Douglas and Canelo were already aware of their guaranteed millions before the match even in case of a loss, they were obviously less prepared and lost the match.

Proposition 2: *If the champion receives less (more) than the threshold level offered by the challenger, then the challenger is willing to put more (less) efforts to win the fight for additional stake to the champ.*

Notice that $\boldsymbol{\beta < \frac{\alpha}{2}}$ implies $\boldsymbol{1-\beta > 1-\frac{\alpha}{2} > \frac{\alpha}{2}}$. In words, if the challenger is guaranteed to receive more than $\boldsymbol{1-\frac{\alpha}{2}}$, then this amount gives him more incentive to put in huge efforts. That is, $\boldsymbol{\frac{\partial e_2^*}{\partial(1-\beta)} > 0}$. If not, then the opposite is true. Comparing the outcomes of equations [8] and [9] we can summarize the fighters' efforts in the Table-1 below.

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[7] In this paper we are not considering a dynamic model provided the goal set.

Table-1: Incentives and disincentives of the fighters.

for $\boldsymbol{\beta < \frac{\alpha}{2}}$	$\boldsymbol{\frac{\partial e_1^*}{\partial \beta} > 0}$	$\boldsymbol{\frac{\partial e_2^*}{\partial \beta} > 0}$ or $\boldsymbol{\frac{\partial e_2^*}{\partial (1-\beta)} < 0}$
for $\boldsymbol{\frac{\alpha}{2} < \beta < \frac{1+\alpha}{2}}$	$\boldsymbol{\frac{\partial e_1^*}{\partial \beta} > 0}$	$\boldsymbol{\frac{\partial e_2^*}{\partial \beta} < 0}$ or $\boldsymbol{\frac{\partial e_2^*}{\partial (1-\beta)} > 0}$
for $\boldsymbol{\beta > \frac{1+\alpha}{2}}$	$\boldsymbol{\frac{\partial e_1^*}{\partial \beta} < 0}$	$\boldsymbol{\frac{\partial e_2^*}{\partial \beta} < 0}$ or $\boldsymbol{\frac{\partial e_2^*}{\partial (1-\beta)} > 0}$

Table-1 yields us three different scenarios. Let's start discussing with two extreme cases first, where moral hazard theoretically holds.

Case 1, $\boldsymbol{\beta < \frac{\alpha}{2}}$, implies that if the share of the fighters increases within the range of $(\boldsymbol{0, \frac{\alpha}{2}})$, for the champ (challenger) it is too low (high). Hence, the additional share from the bargaining will create an incentive (disincentive) for the champ (challenger) to train hard.

Case 3, $\boldsymbol{\beta > \frac{1+\alpha}{2}}$, in principle, is the opposite scenario in Case 1. That is, the additional share from the bargaining will create an incentive (disincentive) for the challenger (champ). Realistically, the likelihood of Case 1 (moral hazard action by the challenger) should be low especially if the champ is an absolute title-holder like Canelo, who would never agree for such a low share while the challenger, on the contrary, should be hungry for the absolute championship belt. However, the opposite scenario, which is Case 3 (moral hazard action by the champ), is more usual. That is, the champ, who is already a Big Name, once guaranteed presumably a big enough share from the bargaining to financially support his future generations, will most likely put less effort in training for a small increase in money share. This is especially true when the champ's career is close to retirement. On the other hand, for the challenger, this should be a great motivation to put huge efforts to win the fight, despite the low share from the bargaining like it happened to Bivol (then less popular) against Canelo (big name) in May 2022.

Finally, Case 2 implies that if the share of the fighters increases within the range of $(\boldsymbol{\frac{\alpha}{2}, \frac{1+\alpha}{2}})$, both the champ and the challenger are willing to put in huge efforts to win the fight. This is common when the two fighters are worth each other so that none of them is successful at receiving their threshold level from negotiation. For ease of reference, the ranges of shares from bargaining for moral hazard are also depicted in Figure 1 below.

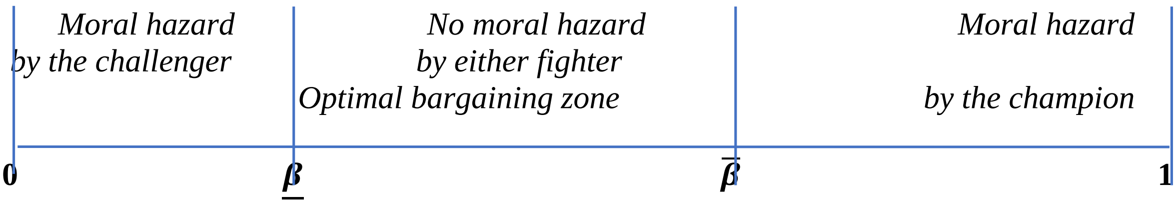

Figure 1: Moral hazard zone

Plugging [6] and [7] into the money function we obtain:

$$\boldsymbol{M(\alpha, \beta) = [\alpha\beta]^{\alpha}[(1-\alpha)(1-\beta)]^{1-\alpha}} \qquad [\mathbf{10}]$$

Now, from stage 1 the optimal share of money revenue for each player is received according to Nash bargaining as:

$$\boldsymbol{\max_{\beta}[\beta M(\alpha, \beta) - d_1][(1-\beta)M(\alpha, \beta) - d_2]} \qquad [\mathbf{11}]$$

From the FOC in [11] we receive:

$$\left(M+\beta\frac{\partial M}{\partial\beta}\right)[(1-\beta)M-d_2]=\left(M-(1-\beta)\frac{\partial M}{\partial\beta}\right)[\beta M-d_1] \qquad [12]$$

Taking the derivative of money revenue with respect to β in [10], which is $\frac{\partial M}{\partial\beta}=\frac{M(\alpha-\beta)}{\beta(1-\beta)}$, and plugging it into [12] will further simplify to:

$$\beta(1+\alpha-2\beta)[(1-\beta)M-d_2]=(1-\beta)(2\beta-\alpha)[\beta M-d_1] \quad [13]$$

Though there is no closed-form solution for optimal shares from the Nash bargaining in [13], we can implicitly say that $\beta^*=\beta^*(\alpha,d_1,d_2)$**.** The fact that the optimal share of each fighter doesn't depend on his respective or their overall efforts, but rather on share of money revenue collected from their respective fans and the outside options is consistent with reality. Since the main goal of the current paper is to determine how the efforts of fighters respond to additional stakes, we are not concerned about the computation of the money shares. However, we can derive the following comparative statics analyses (See **Appendix**)

(A.1). $\frac{\partial\beta^*}{\partial\alpha}>0$

(A.2). $\frac{\partial\beta^*}{\partial d_1}>0$

(A.3). $\frac{\partial\beta^*}{\partial d_2}<0$

Normally, in the pro boxing industry it is common that the higher the money contribution by the fighters' fans, the higher is his share from bargaining against his opponent. Hence, the first condition sounds intuitive. The last two are consistent with Nash bargaining literature. Hence, the additional outside option of the champ (challenger) has a positive (negative) impact on the champion's share.

Notice for the equilibrium share of the fighters from bargaining in [13] to hold, either of the following two conditions must satisfy.

(C.1). $\underline{\beta}\le\beta^*\le\overline{\beta}$ (Case 2)

(C.2). $\underline{\beta}\ge\beta^*\ge\overline{\beta}$ (Case 1 and Case 3)

Mathematically, the condition **(C.2)** is counter-intuitive since $\underline{\beta}=\frac{\alpha}{2}<\overline{\beta}=\frac{1+\alpha}{2}$. Then the first condition **(C.1)** means that the optimal share for each fighter from the bargaining must be designed such that neither of the fighters should act in an unusual way to cause a moral hazard. That is, $\beta^*\epsilon\,(\underline{\beta},\overline{\beta})$[8]**.**

Proposition 3: *For the optimal share for each fighter from the bargaining to hold, there must be an incentive for both fighters to put huge efforts for an additional stake.*

In simple words, proposition 3 states that in case at least one fighter demonstrates a disincentive for an additional stake of money revenue, the bargaining between the two fighters on money share breaks down.

Plugging the optimal share from [13] into [6] and [7], the optimal efforts of the fighters become:

$$e_1^*=e_1^*(\alpha;\ \beta^*(\alpha,d_1,d_2)) \qquad [14]$$

$$e_2^*=e_2^*(\alpha;\ \beta^*(\alpha,d_1,d_2)) \qquad [15]$$

The shares of money revenue collected from the fans have both direct and indirect impacts on fighters' efforts. That is,

[8] Note the range does not necessarily imply $\beta_L=\underline{\beta}$ or $\beta_H=\overline{\beta}$. So if $\underline{\beta}<\beta_L<\beta_H<\overline{\beta}$**,** then there won't be a problem of moral hazard and optimal bargaining will be reached. But if *either* $\beta_L<\beta^*<\underline{\beta}$ *or* $\overline{\beta}<\beta^*<\beta_H$**,** then moral hazard is unavoidable.

$$\frac{\partial e_i^*}{\partial \alpha} = \frac{de_i^*}{d\alpha} + \left(\frac{\partial e_i^*}{\partial \beta^*}\right)\left(\frac{\partial \beta^*}{\partial \alpha}\right) \quad for\ i = 1, 2 \qquad [16]$$

So, the sign of the equation [16] depends on the signs of three terms on the RHS. From equation [6] it is not difficult to show the direct impact of $\boldsymbol{\alpha}$ on champion's effort as:

$$\frac{de_1^*}{d\alpha} = \frac{e_1^*}{2}\left[log\left(\frac{\alpha}{\{1-\alpha\}^{\alpha}}\right) + \frac{2+\alpha}{2\alpha}\right] > 0 \qquad [17]$$

Assuming the successful bargaining in the first stage with the absence of moral hazard by each fighter, the increase in the shares of the money revenue collected from the fans of champ's motivates the champ to put huge efforts to make them happy. That is,

$$\frac{\partial e_1^*}{\partial \alpha} = \underbrace{\frac{de_1^*}{d\alpha}}_{\substack{>0 \\ by\,[17]}} + \underbrace{\left(\frac{\partial e_1^*}{\partial \beta^*}\right)}_{\substack{>0 \\ no\ moral \\ hazard}} \underbrace{\left(\frac{\partial \beta^*}{\partial \alpha}\right)}_{\substack{>0 \\ by \\ (A.1)}} > 0 \qquad [18]$$

The outside options of each fighter have an indirect impact on their efforts. For the case of champ:

$$\frac{\partial e_1^*}{\partial d_1} = \underbrace{\left(\frac{\partial e_1^*}{\partial \beta^*}\right)}_{\substack{>0 \\ no\ moral \\ hazard}} \underbrace{\left(\frac{\partial \beta^*}{\partial d_1}\right)}_{\substack{>0 \\ by \\ (A.2)}} > 0 \qquad [19]$$

$$\frac{\partial e_1^*}{\partial d_2} = \underbrace{\left(\frac{\partial e_1^*}{\partial \beta^*}\right)}_{\substack{>0 \\ no\ moral \\ hazard}} \underbrace{\left(\frac{\partial \beta^*}{\partial d_2}\right)}_{\substack{<0 \\ by \\ (A.3)}} < 0 \qquad [20]$$

Proposition 4: *With the absence of moral hazard by each fighter, the additional outside option of the champ (challenger) will create an incentive (disincentive) for the champ to put more efforts into training.*

Proposition 4 is intuitive and straightforward. Another name for the outside options of the fighters are their opportunity costs of the current match. So the huge opportunity cost of the champion improves his bargaining power against the challenger, which contributes to a larger share to the champion. This, in turn, motivates the champ to exert more effort. Similarly, the large opportunity cost of the challenger leads to a lower bargaining power of the champ and reduces the champ's share. Thus, the opportunity cost of the challenger doesn't encourage the champ to train properly increasing the likelihood of a poor showing.

From equation [7] it is not difficult to show the direct impact of $\boldsymbol{\alpha}$ on challenger's efforts as:

$$\frac{\partial e_2^*}{\partial \alpha} = \frac{e_2^*}{2}\left[log\left(\frac{\alpha}{\{1-\alpha\}^{\alpha}}\right) - \frac{1}{1-\alpha}\right] < 0 \qquad [21]$$

Then from the comparative statics of efforts by the challenger in equation [15] we receive [22]-[24]:

$$\frac{\partial e_2^*}{\partial \alpha} = \underbrace{\frac{de_2^*}{d\alpha}}_{\substack{<0 \\ by\,[21]}} + \underbrace{\left(\frac{\partial e_2^*}{\partial \beta^*}\right)}_{\substack{<0 \\ no\ moral \\ hazard}} \underbrace{\left(\frac{\partial \beta^*}{\partial \alpha}\right)}_{\substack{>0 \\ by \\ (A.1)}} < 0 \qquad [22]$$

The equation [22] implies that if the share of money revenue collected from the fans of the champ increases, then it demotivates the challenger to put huge efforts in preparation for the forthcoming fight. This could be due to the fact that usually the magnitude of the bargaining outcomes for each boxer depends on the cash contribution of their respective fans. Then the greater contribution of the champ's fans may imply the greater share of money revenue would go to the champ that would discourage the challenger.

$$\frac{\partial e_2^*}{\partial d_1} = \underbrace{\left(\frac{\partial e_2^*}{\partial \beta^*}\right)}_{\substack{<0 \\ no\ moral \\ hazard}} \underbrace{\left(\frac{\partial \beta^*}{\partial d_1}\right)}_{\substack{>0 \\ by \\ (A.2)}} < 0 \qquad [23]$$

$$\frac{\partial e_2^*}{\partial d_2} = \underbrace{\left(\frac{\partial e_2^*}{\partial \beta^*}\right)}_{\substack{<0 \\ no\ moral \\ hazard}} \underbrace{\left(\frac{\partial \beta^*}{\partial d_2}\right)}_{\substack{<0 \\ by \\ (A.3)}} > 0 \qquad [24]$$

The outcomes in [23] and [24] are symmetric versions of [19] and [20].

3. Concluding remarks

In this paper we built a theoretical model on the linkages between negotiated money share of the professional boxers and their corresponding efforts afterwards put during the training, which was implicitly mentioned in the existing literature, yet hasn't been investigated. On the one hand, any monetary benefit given to fighters should extrinsically incentivize them to perform to the best of their abilities to win the fight. On the other hand, financial motivation can also create a disincentive for the fighters once they are fully insured with a large enough guaranteed purse regardless of the upcoming fight's outcome. Hence, the main goal of the paper has been to understand how the pro boxers react when their money share from the bargaining increases.

The theoretical findings in this paper show that the additional money share was found to motivate fighters to put huge efforts under a certain threshold, and that above the threshold, the relationship was reversed which refers to the moral hazard phenomenon. More important novelty is that for a successful bargaining between the fighters on money share to hold, the range (or threshold) of the money shares for each fighter must be designed in such a way that for a small change of stake to either fighter shouldn't alter their behavior (efforts). We also found that the opportunity costs of the fighters have an indirect impact on their efforts via their negotiated shares. That is, the opportunity cost of the fighter has a positive impact on his efforts while that of the opponent has a negative impact on his efforts.

There are some important issues not covered in this paper considering the growing field of sports economics.

1. In reality, as was mentioned in Issabayev and Oskenbayev (2019), the bargaining on money share should be held between the promoters/managers on behalf of their respective fighters. Hence, the money revenue itself should be a function of the efforts of the whole team of the fighters be it promoters, managers, doctors, coaches etc. Unfortunately, for tractability reasons to get some insights of the story we have instead maintained the assumption of money revenue to be a function of the fighters' efforts only. Though the value parameters V_1 and V_2 were implicitly capturing the worth of all the members in the team.
2. One way to do it is to incorporate a principal-agent problem between the promoters/managers and fighters to ensure that the fighters (as agents) would choose their efforts in training subject to no moral hazard condition. Or tools of mechanism design are worth trying for this scenario as well.
3. Another way to extend this topic is to try a dynamic (two-period) game counting both past and contemporaneous efforts of the fighters, so that the past performance would build a reputation and

test how it would influence current efforts once the optimal share from bargaining is predetermined based on past performance.

4. It also should be interesting to analyze the fighters' behavior before and after the bargaining. Their efforts could be different or constant depending on the amount. For before the bargaining starts they choose their efforts under uncertainty while after the bargaining is over they choose efforts under certainty.

5. Moreover, there is a possibility that the moral hazard issue may arise even from a small amount of share as long as the fighter's money revenue covers his outside options. In other words, the fighters are to receive their purse upon negotiation regardless of win or loss. Imagine the fighter is guaranteed 25% of total money revenue after the fight ends and this is the largest amount that he cannot earn elsewhere. Even this small amount may cause a fighter not to exert the proper effort level. For he knows exactly that this 25% of revenue won't change even if he wins. Hence, one may search for anecdotes for this scenario and build respective models.

6. In addition to the last point, one may try to develop a theoretical model where the optimal share of money revenue from bargaining is subject to change upon the fight's outcome. Suppose the initial amount of share from bargaining for a fighter is "$\boldsymbol{s}$". If he wins the fight, his share is to increase by, say $\boldsymbol{\delta > 0}$, so that ex post he is expected to receive "$\boldsymbol{s + \delta}$". Similarly, in case of loss his share will reduce by $\boldsymbol{\delta}$**,** that is, he is expected to receive "$\boldsymbol{s - \delta}$". This in practice, to the best of our knowledge, doesn't seem to work. However, from management perspective, it would be a useful policy for promoters in order, at least, to avoid the moral hazard problem by fighters ex ante.

These issues should definitely be interesting to consider for future research. Hence, we hope in the future the preliminary outcomes in the current paper will be extended in a number of ways.

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REFERENCES

1 Akin, Z., Issabayev, M., Rizvanoghlu, I. (2022) "Incentives and Strategic Behavior of Professional Boxers", *forthcoming at Journal of Sports Economics*.

2 Amegashie, J.A., Kutsoati, E. (2005) "Rematches in boxing and other sporting events", *Journal of Sports Economics*, 6(4), 401 – 411.

3 Butler D., Butler R., Maxcy J. (2020) "New insights on the Louis-Schmeling paradox: Determinants of demand for subscription and pay-per-view boxing", *European Sport Management Quarterly*, 1 – 21.

4 Chaplin, D. (2012) "Pay and Race in World Championship Boxing", *Journal of Economics*, 38(1), 43 – 57.

5 Chaplin D., Madrigal E., Magueta D. (2017) "Outcome uncertainty, the superstar effect, talent and live attendance in professional boxing", Athens Journal of Sports, 4(3), 197 – 212.

6 Issabayev M., Oskenbayev, Y. (2019) "Title or Money", *Economics Bulletin*, 39(3), 1912 – 1917.

7 Nash, J. (1953) "Two-Person Cooperative Games", *Econometrica* **21**, 128 – 140.

8 Tenorio, R. (2000) “The economics of professional boxing contracts”, *Journal of Sports Economics*, 1(4), 363 – 384.

9 Tenorio, R. (2006) “On the competitive structure in professional boxing, or why the best boxers very seldom fight each other”, *Handbook on the Economics of Sport*, 364 – 368.

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ОҢТАЙЛЫ КЕЛІССӨЗДЕР ЖӘНЕ МОРАЛЬДЫҚ ҚАУІП

Аңдатпа. Бұл мақалада біз кәсіпқой боксшылардың бұрынғы лауазымда өзін қалай ұстайтыны туралы сұраққа жауап табуға тырысамыз. Біз белгілі бір ставка шегінде тұрған әрбір жауынгер үшін алдағы жекпе-жекке үлкен күш салуға ынталандыру бар екенін және бұл шектен жоғары моральдық қауіп құбылысына әкелетін керісінше шындық екенін анықтаймыз. Біз сондай-ақ моральдық қауіп болмаған жағдайда келіссөздер сәтті өтетінін байқаймыз. Ақырында, жауынгердің оңтайлы күш-жігері оның (қарсыластарының) қосымша сыртқы нұсқасы үшін артады (азаяды).

Түйін сөздер: келіссөздер, бокс, күш-жігер, ынталандыру, моральдық қауіп, шекті үлес.

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ОПТИМАЛЬНЫЙ ТОРГ И МОРАЛЬНЫЙ РИСК

Абстракт. В этой статье мы попытаемся найти ответ на вопрос о том, как ведут себя профессиональные боксеры после получения дополнительной денежной доли, оговоренной заранее. Мы обнаружили, что у каждого бойца при определенном уровне ставок есть стимул приложить огромные усилия для предстоящего боя, а при превышении этого порога все наоборот, что приводит к возникновению морального риска. Мы также обнаружили, что переговоры успешны при отсутствии морального риска. Наконец, оптимальное усилие бойца будет увеличиваться (уменьшаться) для его (его противников) дополнительного внешнего варианта.

Ключевые слова: переговоры, бокс, усилия, стимулы, моральный риск, пороговая доля.

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Appendix

The first-order condition (FOC) in [13] can be written in implicit form as

$$\boldsymbol{F(\beta,\alpha,d_1,d_2) \equiv \beta(1+\alpha-2\beta)[(1-\beta)M-d_2]-(1-\beta)(2\beta-\alpha)[\beta M-d_1]=0}$$

with $\boldsymbol{1+\alpha-2\beta>0}$ and $\boldsymbol{2\beta-\alpha>0.}$ Then, the second-order condition (SOC) can be computed as:

$$\boldsymbol{F_\beta \equiv (1+\alpha-2\beta)[(1-\beta)M-d_2]-2\beta[(1-\beta)M-d_2]+\beta(1+\alpha-2\beta)\left[-M+\beta\frac{\partial M}{\partial \beta}\right]}$$
$$\boldsymbol{+(2\beta-\alpha)[\beta M-d_1]-2(1-\beta)[\beta M-d_1]-(1-\beta)(2\beta-\alpha)\left[M+\beta\frac{\partial M}{\partial \beta}\right]}$$

Using $\boldsymbol{\frac{\partial M}{\partial \beta}=\frac{M(\alpha-\beta)}{\beta(1-\beta)}}$ with few algebra, we receive SOC as:

$$\boldsymbol{F_\beta \equiv (1+\alpha-4\beta)[(1-\beta)M-d_2]-2M(2\beta-\alpha)(1+\alpha-2\beta)}$$
$$\boldsymbol{-[\beta M-d_1](2+\alpha-4\beta)}$$

The first term on the RHS is negative if and only if $\boldsymbol{\beta \geq \frac{1}{2}}$, while the second term is obviously positive. The sign of the third term is ambiguous. For convenience we assume $\boldsymbol{F_\beta<0}$.

$$\boldsymbol{F_\alpha \equiv \beta[(1-\beta)M-d_2]+\beta(1+\alpha-2\beta)\left[(1-\beta)\frac{\partial M}{\partial \alpha}\right]+(1-\beta)[\beta M-d_1]}$$
$$\boldsymbol{-(1-\beta)(2\beta-\alpha)\left[\beta\frac{\partial M}{\partial \alpha}\right]}$$

From equation [10] $\boldsymbol{\frac{\partial M}{\partial \alpha}=Mlog\left[\frac{\alpha\beta}{(1-\alpha)(1-\beta)}\right]>0.}$ Plugging it into the last equation we receive:

$$\boldsymbol{F_\alpha \equiv \beta[(1-\beta)M-d_2]+(1-\beta)[\beta M-d_1]+\beta(1-\beta)\left[\frac{\partial M}{\partial \alpha}\right](1+2\alpha-4\beta)}$$

The first two terms on the RHS are positive while the sign of the last term is dubious. For convenience, we assume $\boldsymbol{F_\alpha>0.}$

$$\boldsymbol{F_{d_1} \equiv (1+\beta)[2\beta-\alpha]>0}$$
$$\boldsymbol{F_{d_2} \equiv -\beta[1+\alpha-2\beta]<0}$$

Hence, by Implicit Function Theorem we obtain:

$$\boldsymbol{\frac{\partial \beta^*}{\partial \alpha}=-\frac{F_\alpha}{F_\beta}=(-)\frac{(+)}{(-)}>0 \quad (A.1.)}$$
$$\boldsymbol{\frac{\partial \beta^*}{\partial d_1}=-\frac{F_{d_1}}{F_\beta}=(-)\frac{(+)}{(-)}>0 \quad (A.2.)}$$
$$\boldsymbol{\frac{\partial \beta^*}{\partial d_2}=-\frac{F_{d_2}}{F_\beta}=(-)\frac{(-)}{(-)}<0 \quad (A.3.)}$$